Spatial-temporal Gauss-Laguerre waves in dispersive media

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A family of dispersionless and diffractionless spatial-temporal Gauss-Laguerre waves propagating in dispersive linear and transparent media is introduced. Contrary to pulsed Bessel beams and envelope-X waves recently studied in media with normal dispersion, these spatiotemporal Gauss-Laguerre beams may exist both in the normal and anomalous dispersion spectral regions of the medium.

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I. INTRODUCTION

Spatial-temporal localization of acoustic or electromagnetic waves capable of propagating undistorted in vacuum, such as X waves, focus wave modes, pulsed Bessel beams, etc., have been the subject of an intense research in the past few years (see, e.g., Refs. [1–9], and references therein), and special attention has been payed to their unusual properties, related to the superluminal [6] or subluminal [8] propagative nature, to the construction of finite energy solutions (see, e.g., Ref. [9]), and to their practical implementation (see, e.g., Ref. [10]). Later investigations have predicted the existence of localized envelope light waves propagating without spreading both in space and time in linear dispersive and transparent media as a result of spatial-temporal coupling effects [11–17]. The potential interest of such localized waves for applications in optical communications, metrology, spectroscopy, and imaging was also pointed out. Recently, some of these localized waves have been shown to play an important role in nonlinear optical processes as well [18–20]. The existence of localized propagating light waves in dispersive linear media which do not show spreading effects both in space and time was first predicted in Ref. [11], where localized waves with axial symmetry were constructed as a superposition of monochromatic Bessel beams with a frequency-dependent cone angle. By specializing the general form of polychromatic Bessel beams of Ref. [11], recent studies have further introduced special nondiffracting and nondispersive *envelope* wave solutions, such as pulsed Bessel beams [13,15] and luminal envelope-X waves [16]. For such waves the mechanism underlying cancellation of temporal spreading due to dispersion and spatial spreading due to diffraction is possible solely in the normal dispersion spectral region of the material. For many applications, such as in optical communications, the optical waves fall however in the anomalous dispersion region of the material, and it would be desirable to highlight a mechanism for wave localization independent of the sign of second-order group velocity dispersion of the medium. Though it is commonly believed that angular dispersion of polychromatic Bessel beams can only compensate for normal material dispersion, it has been very recently shown that material group velocity dispersion cancellation can occur in the anomalous spectral region as well in a special class of polychromatic Bessel beams [17].

In this paper we show the existence of a family of axially symmetric propagation-invariant linear localized waves in dispersive materials that may exist both in the normal and in the anomalous dispersion spectral regions of the material. The spatial-temporal wave envelope is expressed in terms of Gauss-Laguerre functions and propagates almost at a luminal group velocity. The paper is organized as follows. In Sec. II the basic grounds on optical wave propagation in dispersive media are reviewed, and both an integral and differential representation of dispersionless and diffractionless wave envelopes is derived. In Sec. III the family of axially invariant Gauss-Laguerre spatial-temporal beams is introduced, and some numerical results are presented. Finally, in Sec. IV the main conclusions are outlined.

II. DISPERSIONLESS AND DIFFRACTIONLESS OPTICAL WAVE ENVELOPES IN DISPERSIVE TRANSPARENT MEDIA

A. Integral representation

We start our analysis by considering optical wave propagation in a linear and transparent medium far from resonances, with a real-valued refractive index that varies with frequency, $n=n(\omega)$. The most general solution to the scalar wave equation for the electric field $E(x,y,z,t)$ is given by the superposition of monochromatic plane waves at frequency ω and wave vector $\mathbf{k}=(k_x,k_y,k_z)$ satisfying the dispersion relation $|\mathbf{k}|=k(\omega)=\omega n(\omega)/c_0$, where c_0 is the speed of light in vacuum, i.e., one has

$$E(x,y,z,t)=\int d\omega dk_x dk_y dk_z \hat{E}(k_x,k_y,k_z,\omega)\,\delta(k_x^2+k_y^2+k_z^2-k^2(\omega))\exp[i\omega t-i(k_x x+k_y y+k_z z)]+\text{c.c.} \tag{1}$$

In Eq. (1), $\hat{E}(k_x,k_y,k_z,\omega)$ is the spectral amplitude of plane waves, and the integral is extended over the positive-frequency part of the spectrum and to real-valued wave vectors $\mathbf{k}$. Equation (1) describes a propagating nondispersive and nondiffracting *envelope* wave, provided that the longitudinal wave number k_z is chosen to be a *linear* function of frequency ω. Introducing a reference frequency ω_0 (carrier frequency) and setting $k_z(\omega)=k_{z0}+k'_{z0}(\omega-\omega_0)$, where k_{z0} and k'_{z0} are undetermined parameters at this stage, one can formally write

$$E(x,y,z,t)=\exp[i(\omega_0 t-k_{z0}z)]\psi(x,y,\tau)+\text{c.c.}, \qquad (2)$$

where $\tau=t-k'_{z0}z$ is a retarded time and $\psi(x,y,\tau)$ is the wave envelope, given by

$$\psi(x,y,\tau)=\int d\Omega dk_x dk_y \hat{\psi}(k_x,k_y,\Omega)\,\delta(k_x^2+k_y^2-k_\perp^2(\Omega))\exp[i\Omega\tau-i(k_x x+k_y y)], \qquad (3)$$

where we have set

$$k_\perp^2(\Omega)\equiv k^2(\omega_0+\Omega)-(k_{z0}+k'_{z0}\Omega)^2. \qquad (4)$$

The electric field is then expressed by the product of a carrier sinusoidal wave at frequency ω_0, propagating with a phase velocity $v_f=\omega_0/k_{z0}$, and an envelope ψ that propagates without distortions, both in space and time, with a group velocity $v_g=1/k'_{z0}$. A particularly important case, which has been considered in previous works [11,13,14,16], is that of wave envelopes with axial symmetry, i.e., $\psi=\psi(r,\tau)$, where $r=(x^2+y^2)^{1/2}$. In this case the envelope spectral amplitude $\hat{\psi}$ depends on k_x and k_y through $k_x^2+k_y^2$, and the δ-Dirac term in Eq. (3) can be removed after performing the integration with respect to k_x and k_y in cylindrical coordinates. Recalling the integral representation of J_0 Bessel function, one obtains

$$\psi(r,\tau)=\int d\Omega \hat{S}(\Omega)J_0(k_\perp(\Omega)r)\exp(i\Omega\tau), \qquad (5)$$

where $\hat{S}(\Omega)$ is an arbitrary spectral amplitude. To avoid the occurrence of evanescent waves, the integral in Eq. (5) has to be limited to the frequencies for which $k_\perp(\Omega)$ is real valued. In particular, for near-monochromatic waves, the expression of $k_\perp(\Omega)$ can be simplified after a power expansion of $k(\omega_0+\Omega)$ around ω_0. By pushing the power expansion up to second order in Ω to account for group-velocity dispersion effects, one has

$$k_\perp^2(\Omega)=A+B\Omega-C\Omega^2, \qquad (6)$$

where we have set

$$A\equiv k_0^2-k_{z0}^2, \qquad (7)$$

$$B\equiv 2(k_0k'_0-k_{z0}k'_{z0}), \qquad (8)$$

$$C\equiv k'^2_{z0}-k'^2_0-k_0k''_0, \qquad (9)$$

and $k_0\equiv k(\omega_0)$, $k'_0\equiv(\partial k/\partial\omega)_{\omega_0}$, $k''_0\equiv(\partial^2k/\partial\omega^2)_{\omega_0}$. Note that, in the case of propagation in vacuum, Eq. (6) is rigorous with $k''_0=0$ in Eq. (9).

B. Differential equation for the wave envelope

Instead of using the integral representation (3) of the wave envelope ψ, it is worth writing down also a differential equation for ψ. To this aim, let us observe that, due to the δ-Dirac term entering in Eq. (3), only the terms satisfying the condition $k_x^2+k_y^2=k_\perp^2(\Omega)$ contribute to the integral, so that one has

$$\int d\Omega dk_x dk_y[k_x^2+k_y^2-k_\perp^2(\Omega)]\hat{\psi}(k_x,k_y,\Omega)\,\delta(k_x^2+k_y^2-k_\perp^2(\Omega))\exp[i\Omega\tau-i(k_xx+k_yy)]=0. \qquad (10)$$

Since

$$\int d\Omega dk_x dk_y(k_x^2+k_y^2)\hat{\psi}(k_x,k_y,\Omega)\,\delta(k_x^2+k_y^2-k_\perp^2(\Omega))\times\exp[i\Omega\tau-i(k_xx+k_yy)]=-\nabla_\perp^2\psi, \qquad (11)$$

where $\nabla_\perp^2\equiv\partial^2/\partial x^2+\partial^2/\partial y^2$ is the transverse Laplacian, and

$$\int d\Omega dk_x dk_y k_\perp^2(\Omega)\hat{\psi}(k_x,k_y,\Omega)\,\delta(k_x^2+k_y^2-k_\perp^2(\Omega))\times\exp[i\Omega\tau-i(k_xx+k_yy)]=k_\perp^2\left(-i\frac{\partial}{\partial\tau}\right)\psi, \qquad (12)$$

where the operator $k_\perp^2(-i\partial/\partial\tau)$ is defined through the power series expansion of $k_\perp^2(\Omega)$ after the substitution $\Omega\rightarrow -i\partial/\partial\tau$, from Eqs. (10)–(12) one obtains

$$\nabla_\perp^2\psi+k_\perp^2\left(-i\frac{\partial}{\partial\tau}\right)\psi=0. \qquad (13)$$

Equation (13) is the basic partial differential equation that has to be satisfied by the envelope ψ of a dispersionless and diffractionless wave (see also Ref. [16]). In particular, for narrow-band pulses, for which the power expansion (6) for $k_\perp^2(\Omega)$ holds, Eq. (13) takes the simplified form

$$\nabla_\perp^2\psi+A\psi-iB\frac{\partial\psi}{\partial\tau}+C\frac{\partial^2\psi}{\partial\tau^2}=0, \qquad (14)$$

where the coefficients A, B, and C are given by Eqs. (7)–(9). Note that Eq. (5) represents the most general solution to Eq. (13) with cylindrical symmetry, however Eq. (13) [or Eq. (14) in the quasimonochromatic case] is more general and it includes solutions without axial symmetry. The existence and properties of localized solutions to Eq. (14) depend strongly on the values of coefficients A, B, and C given by Eqs. (7)–(9), which are determined once the free parameters k_{z0} and k'_{z0}, which fix the phase and group velocities of the wave, are assigned. As particular cases, Eq. (14) contains the two-dimensional (2D) and 3D elliptic Helmholtz equation (namely, for $B=C=0$, $A>0$ and $B=0$, $A>0$, $C>0$, respectively), the Schrödinger equation (for $A=C=0$, $B\neq 0$), the hyperbolic 2D wave equation (for $A=B=0$, $C<0$), and the 2D Klein-Gordon equation (for $B=0$, $C<0$, and $A<0$).

III. LOCALIZED GAUSS-LAGUERRE WAVES

A. Gauss-Laguerre waves and their Bessel-beam spectral representation

Pulsed Bessel beams [13] and envelope-X waves [16] represent special solutions of the envelope wave equation (14) with axial symmetry in the quasimonochromatic approximation. In fact, one can easily show that these two families of solutions are obtained by imposing $B=C=0$, $A>0$ for pulsed Bessel beams and $A=B=0$, $C<0$ for envelope X waves. For the sake of completeness, these types of solutions are briefly reviewed in the Appendix. What is important to stress here is that these solutions can exist solely for $k_0''>0$, i.e., in the normal dispersion spectral region of the medium. In this section we introduce a family of localized envelope waves with axial symmetry, which can be supported both in the normal and anomalous dispersion regions of the material. As we will show, these localized waves are expressed in terms of Gauss-Laguerre pulsed beams, and propagate undistorted with a group velocity v_g which is almost luminal, i.e., close to the usual value $1/k_0'$. Gauss-Laguerre localized waves are the family of solutions with axial symmetry of the quasimonochromatic envelope equation (14) when the parameters k_{z0} and k_{z0}' are chosen such that $A=C=0$. This occurs by assuming

$$k_{z0}=k_0,\quad k_{z0}'=k_0'\sqrt{1+\frac{k_0k_0''}{k_0'^2}} \tag{15}$$

and, in correspondence, the envelope differential equation reads

$$iB\frac{\partial\psi}{\partial\tau}=\nabla_\perp^2\psi. \tag{16}$$

Note that Eq. (16) is formally equivalent to the paraxial wave equation of diffraction in homogeneous media (see, for instance, Ref. [21]), however the propagation distance is here played by the retarded time $\tau=t-z/v_g$. The expression of the B coefficient in the previous equation is

$$B=2k_0k_0'\left(1-\sqrt{1+\frac{k_0k_0''}{k_0'^2}}\right). \tag{17}$$

The phase velocity of the Gauss-Laguerre waves is given by $v_f=\omega_0/k_0$, whereas the group (envelope) velocity is given by $v_g=1/k_{0z}'\simeq 1/k_0'$ (since usually $|k_0k_0''/k_0'^2|\ll 1$). The sign of the group-velocity dispersion parameter determines the sign of the B coefficient in Eq. (16), namely, one has $B>0$ $[B<0]$ for $k_0''<0$ $[k_0''>0]$. A family of solutions with axial symmetry of Eq. (16) is given by the set of Gauss-Laguerre waves [21], which are explicitly given by

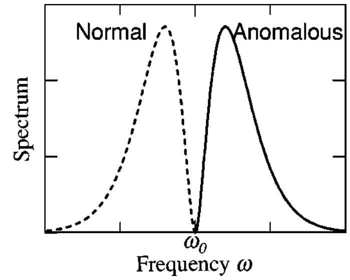

FIG. 1. Qualitative behavior of the spectrum $\hat{S}(\omega)$ of a spatial-temporal Gauss-Laguerre wave (for $n=2$) in the anomalous and normal dispersion regimes.

$$\psi(r,\tau)=\frac{1}{(\tau_0\mp i\tau)^{n+1}}L_n^0\left[\frac{|B|r^2}{4(\tau_0\mp i\tau)}\right]\exp\left[-\frac{|B|r^2}{4(\tau_0\mp i\tau)}\right], \tag{18}$$

where $n=0,1,2,\ldots$, L_n^0 is the generalized Laguerre polynomial of order n, τ_0 is an arbitrary parameter that determines the on-axis pulse duration and transverse beam size, and the upper (lower) sign applies if $B>0$ $(B<0)$. Note that the on-axis $(r=0)$ wave intensity varies with retarded time τ according to

$$|\psi(0,\tau)|^2\propto\frac{1}{(\tau_0^2+\tau^2)^{n+1}}, \tag{19}$$

i.e., it describes a localized pulse with an algebraic decay, and τ_0 determines the pulse duration. Spatial localization is determined mostly by the Gaussian term; in particular, at $\tau=0$, the beam spot size w_0 of the Gaussian beam turns out to be

$$w_0=\sqrt{\frac{4\tau_0}{|B|}}\simeq\sqrt{\frac{4\tau_0k_0'}{k_0^2|k_0''|}}. \tag{20}$$

It is worth considering the spectrum $\hat{S}(\Omega)$ of the Gauss-Laguerre localized waves, which can be derived by means of the Bessel-beam decomposition according to Eq. (5). Indeed, one can show that the Gauss-Laguerre waves, given by Eq. (18), can be obtained from Eq. (5) by setting [22]

$$\hat{S}(\Omega)=\begin{cases}\dfrac{1}{n!}\Omega^n\exp(-\tau_0\Omega), & \Omega>0\\ 0, & \Omega<0\end{cases} \tag{21}$$

for $k_0''<0$, and

$$\hat{S}(\Omega)=\begin{cases}\dfrac{1}{n!}(-\Omega)^n\exp(\tau_0\Omega), & \Omega<0\\ 0, & \Omega>0\end{cases} \tag{22}$$

for $k_0''>0$ (see Fig. 1). Gauss-Laguerre waves thus show a one-side spectrum which is blue shifted (with respect to the reference carrier frequency) in the anomalous dispersion regime $(k_0''<0)$ and redshifted in the normal dispersion regime ($k_0''>0$). It should be noted that the analytic form of Gauss-Laguerre waves given by Eq. (18) is exact if one approximates the dispersion curve $k_\perp(\Omega)$ with its power expansion up to second order in Ω, i.e., by neglecting third-order and higher-order dispersion effects. For ultrashort pulses, i.e., when the spectral extent of $\hat{S}(\Omega)$ is broad, one should consider the integral representation of Gauss-Laguerre waves, given by Eq. (5), after assuming for $k_\perp(\Omega)$ the exact expression given by Eq. (4) and determined by the exact dispersion relation $k(\omega)$ of the material. The existence of Gauss-Laguerre waves requires a nonvanishing group-velocity dispersion parameter, i.e., $k_0''\neq 0$, so that the underlying wave localization mechanism is truly a spatial-temporal effect involving both diffraction and group-velocity dispersion effects. No Gauss-Laguerre waves may exist in vacuum. As a final remark, it should be noted that the condition for group-velocity cancellation achievable with polychromatic Bessel beams in the anomalous dispersion regime recently considered in Ref. [17] leads to a rather different class of localized waves than the Gauss-Laguerre family considered in this work. In fact, if we consider the integral representation of localized diffractionless and dispersionless waves in terms of Bessel beams [Eq. (5)] and introduce the cone angle $\theta(\Omega)$ of Bessel beams according to [17]

$$\sin[\theta(\Omega)]=\frac{k_\perp(\Omega)}{k(\omega_0+\Omega)},\quad \cos[\theta(\Omega)]=\frac{k_z(\Omega)}{k(\omega_0+\Omega)}, \tag{23}$$

the condition for group-velocity cancellation considered in Ref. [17] corresponds to $\theta(\Omega)=\theta_0+\theta_0''\Omega^2/2$, where $\theta_0''=k_0''/(k_0\tan\theta_0)$ and θ_0 is a free-family parameter that determines the phase and group velocities of the localized waves, which are given explicitly by $v_f=\omega_0/(k_0\cos\theta_0)$ and $v_g=1/(k_0'\cos\theta_0)$. In correspondence, using Eqs. (6) and (7) and expanding all terms up to second order in Ω, one obtains $A=k_0^2\sin^2\theta_0$, $B=2k_0k_0'\sin^2\theta_0$, and $C=-k_0'^2\sin^2\theta_0-k_0k_0''$. Since $A\neq 0$ and $C\neq 0$, such polychromatic waves do not correspond to Gauss-Laguerre waves.

B. Numerical examples

In order to provide some numerical examples of Gauss-Laguerre localized waves in dispersive transparent media, let us consider beam propagation in sapphire, which shows a transparent range from $\simeq 300$ nm up to $\simeq 2800$ nm and anomalous dispersion for wavelengths larger than $\simeq 1.3$ μm; similar results can be of course obtained in other transparent dielectric materials, such as fused silica or glasses. In Fig. 2(a) a typical space-time diagram of the envelope intensity $|\psi(r,\tau)|^2$ is shown for a Gauss-Laguerre wave of the second order ($n=2$) at the carrier wavelength (in vacuum) $\lambda_0=2\pi c_0/\omega_0=1.55$ μm of optical communications, where the material dispersion is anomalous [$k_0''\simeq -3.2477\times 10^{-26}$ s^2/rad^2 m, $1/k_0'\simeq 1.6918\times 10^8$ m/s, $n(\omega_0)=1.7462$]. The diagram has been obtained by using the integral representation of Gauss-Laguerre waves in terms of Bessel beams [Eq. (5)] with the exact dispersion curve for $k_\perp(\Omega)$ and assuming a spectral amplitude $\hat{S}(\Omega)$ according to Eq. (21). The dispersion curve $k(\omega)$ for sapphire has been calculated by using a Sellmeier equation for the refractive index according to the data of Ref. [23]. Figure 2(b) shows the behavior of the normalized transverse wave vector $k_\perp/k_0$ as a function of frequency using the Sellmeier equation and compared with the approximate curve given by Eq. (6). The space-time diagram in Fig. 2(a) turns out to be very well fitted by the analytic expression of the Gauss-Laguerre waves given by Eq. (18) [see Fig. 2(c)]. Figure 3 shows the same plots as in Fig. 2 but for a carrier wavelength $\lambda_0=780$ nm, which falls in the normal dispersion region [$k_0''\simeq 6.0098\times 10^{-26}$ s^2/rad^2 m, $1/k_0'\simeq 1.6829\times 10^8$ m/s, $n(\omega_0)=1.7607$]. Figure 4 shows finally the case of a carrier wavelength λ_0

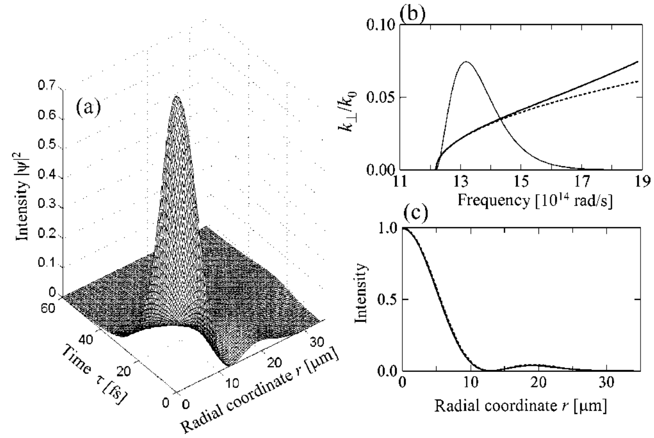

FIG. 2. (a) Space-time plot of envelope intensity $|\psi(r,\tau)|^2$ (in arbitrary units) for a second-order Gauss-Laguerre wave ($n=2$) in sapphire in the anomalous dispersion region ($\lambda_0=1550$ nm). Pulse duration parameter is $\tau_0=20$ fs. (b) Behavior of normalized transverse wave vector $k_\perp/k_0$ vs frequency ω (solid curve); the dashed curve is the corresponding approximate behavior as given by Eq. (6). The shape of the spectral amplitude $\hat{S}(\omega)$ is also shown (thin solid line). (c) Radial behavior of wave intensity $|\psi(r,0)|^2$ at retarded time $\tau=0$ (solid curve) and corresponding approximate curve as given by Eq. (18) (dashed curve, almost overlapped with the solid one).

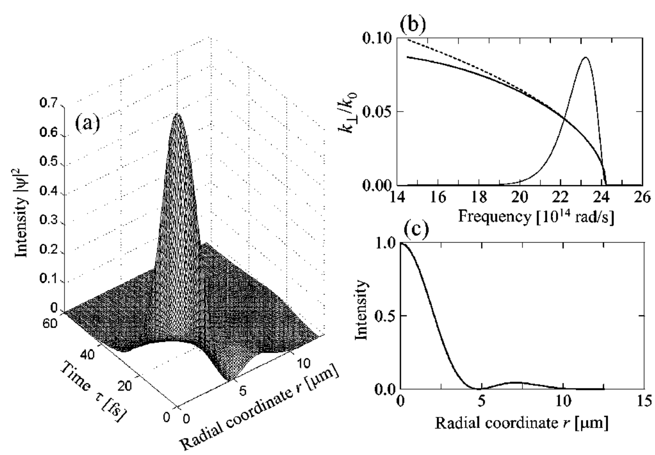

FIG. 3. Same as Fig. 2, but in the normal dispersion region ($\lambda_0=780$ nm). Pulse duration parameter is $\tau_0=20$ fs.

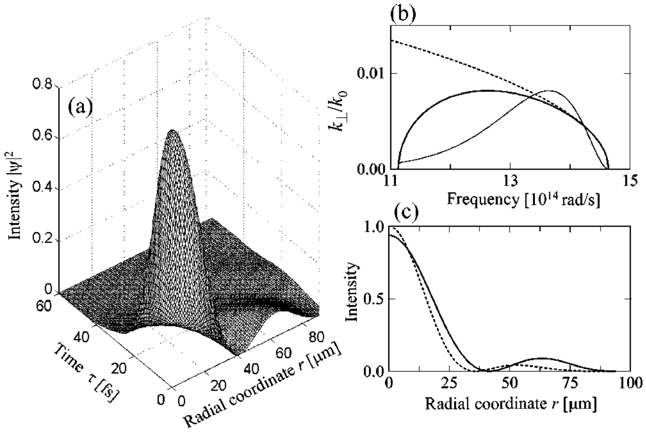

FIG. 4. Same as Fig. 2, but for a carrier wavelength close to the zero group-velocity dispersion point ($\lambda_0=1290$ nm). Pulse duration parameter is $\tau_0=20$ fs.

$=1290$ nm close to the zero dispersion point [$k_0''\simeq 2.9485\times 10^{-27}$ s^2/rad^2 m, $1/k_0'\simeq 1.6927\times 10^8$ m/s, $n(\omega_0)=1.7507$]. Note that, in this case, the behavior of the transverse wave number $k_\perp$ versus frequency differs substantially from the approximate expression obtained by neglecting higher-order dispersion effects. In addition, there exists now a cutoff frequency below which $k_\perp$ becomes purely imaginary. Since Eq. (5) yields a spatially localized wave, provided that $k_\perp$ remains real, the spectrum $\hat{S}(\omega)$ has been truncated and set equal to zero below the cutoff frequency [see Fig. 4(b)]. Note that, due to the large deviation of the dispersion curves in Fig. 4(b), the Gauss-Laguerre wave turns out to remarkably deviate from the analytical expression given by Eq. (18) [see Fig. 4(c)].

IV. CONCLUSION AND DISCUSSION

In this paper we have introduced a class of localized nondiffracting and nondispersive localized waves in linear dispersive transparent media that can be supported both in the normal and anomalous dispersion spectral regions of the material. By neglecting third-order and higher-order dispersion effects, such localized waves satisfy the typical paraxial wave equation of diffraction and are expressed in terms of Gauss-Laguerre functions. The spectral representation of the spatial-temporal Gauss-Laguerre waves in terms of monochromatic Bessel beams has been also presented, and their distinctive features as compared to previously studied pulsed Bessel beams and envelope-X waves have been discussed. As a final comment, it is worth observing that, though most of previous studies on spatiotemporal wave localization have been concerned with solutions showing an axial (radial) symmetry and thus representable as a superposition of monochromatic Bessel beams, the differential approach to the problem of spatial-temporal wave localization in dispersive media, developed in Sec. II, allows one to easily predict the existence of families of waves with broken axial symmetry. For instance, though we have limited our analysis to the radially symmetric solutions to Eq. (16), it is well known that there exist families of solutions with broken axial invariance, such as elliptic Gaussian waves or Gauss-Hermite waves.

APPENDIX: PULSED BESSEL BEAMS AND ENVELOPE-X WAVES

In the case of a material with *normal dispersion* ($k_0''>0$), two types of solutions to Eq. (5) in the near-monochromatic case [or, equivalently, to Eq. (14)] have been recently found in Refs. [13,16], namely, *dispersion-free pulsed Bessel beams* [13] and luminal *envelope-X waves* [16]. The *pulsed Bessel beams* [13] are obtained by assuming $B=C=0$, i.e., $k_{z0}=k_0k_0'/(k_0'^2+k_0k_0'')^{1/2}$ and $k_{z0}'=(k_0'^2+k_0k_0'')^{1/2}$, so that $k_\perp$ is independent of frequency and given by $k_\perp=[k_0^3k_0''/(k_0'^2+k_0k_0'')]^{1/2}$. In this case from Eq. (5) one obtains $\psi(r,\tau)=J_0(k_\perp r)s(\tau)$, where $s(\tau)=\int d\Omega\hat{S}(\Omega)\exp(i\Omega\tau)$ is an arbitrary temporal pulse profile. We thus have a pulsed Bessel beam that propagates without temporal spreading at a group velocity $v_g=1/k_{z0}'\simeq 1/k_0'$ (since in typical cases $|k_0k_0''|\ll k_0'^2$; for more details, see Ref. [13]).

The envelope-X waves [16], propagating at a group velocity $v_g=1/k_0'$, are instead obtained by assuming $A=B=0$, i.e., $k_{z0}=k_0$ and $k_{z0}'=k_0'$. In this case one has $\psi(r,\tau)=\int d\Omega\hat{S}(\Omega)J_0(\sqrt{k_0k_0''}|\Omega|)\exp(i\Omega\tau)$, which yields a typical X-shaped wave in the (r,τ) plane by choosing, e.g., a spectral amplitude $\hat{S}(\Omega)=\exp(-\tau_0|\Omega|)$ (see Ref. [16] for more details). Note that both previous types of localized nondispersive and nondiffracting solutions exist solely in the normal dispersion spectral region.

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